disc 105

Wednesday, March 10, 2021 11:00 AM

4.3 Problems

Problem 1. *Approximate* $\int_0^{.5} \frac{2}{x-4}dx$ *using (a) trapezoidal rule (b) Simpson's rule (c) midpoint rule. Bound the error, and compare that to the actual error.*

Problem 2. *The Trapezoidal rule applied to* $\int_0^2 f(x)dx$ *gives the value 4, and Simpson's rule gives the value 2. What is* $f(1)$*?*

Problem 3. *Find the constants* c_0, c_1, x_1 *so that the quadrature formula* $\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$ *has the highest possible degree of precision.*

1) $\int_a^b f(x)\,dx = \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right) + \frac{h^5}{90} f^{(4)}(\xi)$

$x_0 = a$

$x_1 = \frac{b+a}{2}$ $\quad h = \frac{b-a}{2}$ $\quad \xi \in (a,b]$

$x_2 = b$

2) $4 = \left(f(0) + f(2)\right)$ $\quad *$

$2 = \frac{1}{3}\left(f(0) + 4f(1) + f(2)\right]$ $\quad **$

3. $(**) - *$

$2 = 4f(1)$

$f(1) = \frac{1}{2}$

Any fn s.t.

$f(1) = 1/2$

$f(0) = 4 - f(2)$ free variable

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3) find c_0, x_1, c_1 s.t. $\int_0^1 f(x) = \underbrace{c_0 f(0) + c_1 f(x_1)}_{G(f(x))}$ for all polynomials of degree $\leq K$

where K is as large as possible.

$c_0 f(x_0) + c_1 f(x_1)$

I want to satisfy

$\int_0^1 1\,dx = G(1)$

$\int_0^1 x\,dx = G(x)$

$\int_0^1 x^2\,dx = G(x^2)$

$\vdots \quad \int \quad \vdots$

want!

$1 = \int_0^1 1\,dx = c_0 + c_1$ (1)

$\frac{1}{2} = \int_0^1 x\,dx = c_1 x_1$ (2)

$\frac{1}{3} = \int_0^1 x^2 dx = c_1 x_1^2$ (3)

$x_1 \neq 0$

$c_1 \neq 0$

(3) $\div$ (2) $\boxed{\frac{2}{3} = x_1}$

(2) $c_1 = \frac{1}{2x_1} = \frac{3}{4}$

(1) $c_0 = 1/4$

$\frac{1}{4} = \int_0^1 x^3 dx \overset{?}{=} c_1 x_1^3 = \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)^3$

4.4 Problems

Problem 4. *Determine the values of* n *and* h *required to approximate* $\int_0^2 e^{2x}\sin(3x)dx$ *to within* 10^{-4} *using (a) composite trapezoidal rule (b) composite Simpson's rule (c) composite midpoint rule.*

4.5 Problems

Problem 5. *Use Roomberg integration to compute* $R_{3,3}$ *for* $\int_1^{1.5} x^2 \ln(x)dx$